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The Philosophical Justification for the Equant in Ptolemy's *Almagest*

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Abstract

In this paper I offer a close reading of Ptolemy's philosophical defense of the equant in *Almagest* 9.2. I identify the challenge to the equant that his defense is supposed to meet, characterizing it as a dispute concerning the origin and authority of the astronomer's first principles (ἀρχαί). I argue that the equant could be taken to violate a principle fundamental to the *Almagest's* astronomical project, namely, that the heavenly bodies move only in uniform circular motions. I show that Ptolemy is not unaware of this potential objection, and explore two ways in which he seeks to fend it off.

Keywords

Ptolemy – Aristotle – astronomy – *Almagest* – equant – philosophy and science

1 The *Almagest* and the Equant

With his great treatise, the *Almagest*,[1] the Alexandrian astronomer Ptolemy took up the problem of demonstrating that the anomalous movements of the

1 The Greek name of the treatise was μαθηματικὴ σύνταξις ('Mathematical Composition'). Following convention, I use the Latinized name of the Arabic title (see Pedersen 2011, 15). The text of Ptolemy's *Almagest* used in this paper is that of Heiberg 1898-1903, one volume with two parts (i.1 and i.2). Translations from the *Almagest* are, when stated, from Toomer 1984, with occasional modification; otherwise translations of Greek and Latin passages are my own.

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celestial bodies could be accounted for in terms of uniform circular motions. This project, which had by his time come to be known as that of 'saving the phenomena' (σῴζειν τὰ φαινόμενα), was an old and important one in the history of Greek astronomy: Ptolemy describes it as the guiding problem for astronomers at least since the time of Hipparchus (fl. second century BCE), while later authors were to retroject its origins even to the age of Plato and Eudoxus.[2] Ptolemy singles out Hipparchus for his contributions to the project in the case of the sun and the moon, but surmises that it had proved too 'difficult' (δύσκολον) for him, as it had for all the others, to complete it by providing an explanation for the anomalies of the five planets.

In this respect Ptolemy evidently thought that he had found a solution where those before him had failed,[3] and he presents it in Books Nine and following of the *Almagest*. His solution depends on a novel modification of the traditional epicylic (Figure 1) and eccentric (Figure 2) models of planetary motion: in a word, it consists in the introduction of a third point—distinct from the center of the Earth and from the center of the planetary deferent—with respect to which the motion of the planets would seem uniform (Figure 3).[4] By such a device, Ptolemy could say truthfully, to his mind at least, that he had not only produced an accurate and predictively powerful kinematic model of the universe but also that in so doing he had 'preserved uniform and circular motions in absolutely all cases' (i.2. 212.20-1, κατὰ πάντων ἁπλῶς τὴν ὁμαλὴν καὶ ἐγκύκλιον κίνησιν διασῴζεσθαι).

Later astronomers conceptually reified this third point and, because it served to regularize, or *equalize*, the planetary anomalies, called its circle the

2 See Ptolemy's historical sketch at i.2. 208.12-211.21 with Pedersen 2011, 34-5 (relevant too are the passages cited below p. 426). For the meaning and history of σῴζειν τὰ φαινόμενα, see Duhem 1903 with Lloyd 1978; Mittelstrass 1962; Bowen 2013, 251-9. It is Simplicius (*in De Cael.* vii. 488.18-24 Heiberg) who furnishes the evidence for a Platonic attribution of the project. The possibility is cautiously allowed by e.g. Vlastos 1975, 59-61 and Mueller 1992, but more recently it has been vigorously argued this attribution is an anachronism, and that there is in fact no decisive evidence prior to the second century BCE that the ancients were even aware of the planetary anomalies that Ptolemy would attempt to save: see Goldstein 1997; Bowen 2001; 2002; 2013, 81-2, 230-48, 251-9. *Contra* a Platonic attribution on other grounds, see also Knorr 1990; Zhmud 1998, 217-8.

3 Even if it was the Babylonian tradition of astronomy that suggested to Ptolemy the way out of this problem, it is nevertheless clear from *Alm.* 9.2 and other passages (cited *passim* below) that he wanted his work to be understood within the confines of a project that he locates in early Greek astronomy (i.e. at least with Hipparchus).

4 Although in the case of Mercury the solution is somewhat different from that for the other planets: see concisely Jones 2004, 375. For an introduction to the equant, see Evans 1984.

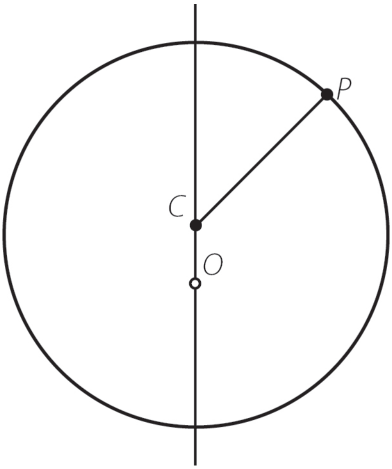

FIGURE 1
The eccentric model: Earth O *is displaced from center of revolution* C*, around which celestial body* P *is borne with uniform angular velocity.*
DIAGRAM: SCOTT WALKER

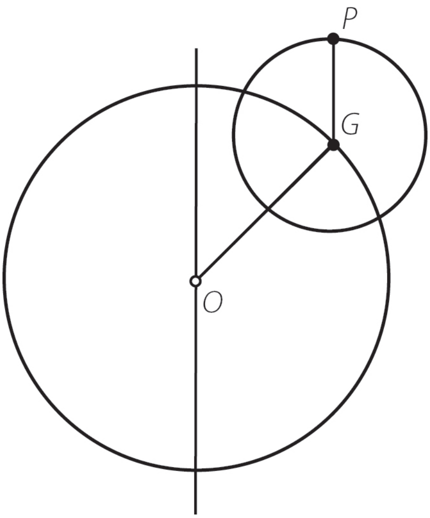

FIGURE 2
The epicyclic model: Earth O *is the center of revolution for center* G *of an epicycle carrying celestial body* P. G *is borne on a deferent circle around* O *with uniform angular velocity.*
DIAGRAM: SCOTT WALKER

(*circulus*) *aequans*, the 'equant' in present-day English. Despite its crucial mathematical function, the equant is connected with an old problem (or perhaps it may be described simply as an old worry) in the *Almagest*. Here I mean the question whether it can be legitimately said to preserve the principle of uniform circular motion in terms of which Ptolemy emphatically describes his astronomical project,[5] or whether in fact it constitutes an unacceptable theoretical departure from such a principle. Famously, Ptolemy's learned Arabic

5 See for example the passages cited below pp. 423, 426.

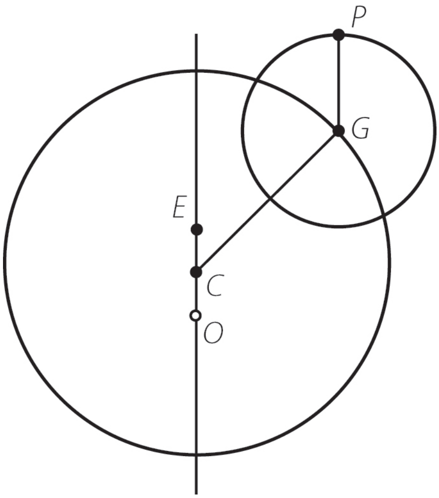

FIGURE 3
The equant: the center G *of an epicycle is borne around* C. *Angular velocity of this revolution is uniform not with respect to* C, *but rather with respect to equant point* E.
DIAGRAM: SCOTT WALKER

commentators and the Renaissance astronomer Copernicus held that the latter was the case, inveighing against the *monstrum* (to quote Copernicus) that it was supposed to introduce into a largely coherent and admirable system.[6] But one need not wait for the next chapter in the history of astronomy to find stirrings of trouble for the equant: evidence internal to the *Almagest* shows that Ptolemy had already considered the complications of introducing the device and that he expected objections to it. Thus, with some evident discomfiture, he asks his readers in *Alm.* 9.2 to pardon him if he is 'compelled' (i.2. 211.23, ἀναγκαζώμεθα)[7] to adopt a mathematical strategy that is 'against theory' (i.2. 211.24, παρὰ τὸν λόγον),[8] and the elaborate and painstaking *apologia* that follows (i.2. 211.21-212.23) cements the impression of Ptolemy's concern about the reception of the equant. While we may be uncertain exactly how his Alexandrian contemporaries would have reacted,[9] this defense reveals Ptolemy anticipating criticism of his solution to the planetary anomalies and

6 For the Arabic astronomers, see Sabra 1999; for Copernicus, see p. 3 of the preface to his *De revolutionibus orbium coelestium* (Nuremburg 1543) with Miller 2014, 27-63.

7 For brief reflections on the logical and persuasive force of ἀναγκάζω in a philosophical context, see Jones 2012.

8 For the full context of this remark, see Section 2 below.

9 The caution of Lloyd 1978, 219 about our ignorance on the matter is *à propos*.

attempting to forestall it. It is important both for its self-consciously philosophical stance and for how it sheds light on Ptolemy's attitude towards the equant.

Recent scholarship on Ptolemy and the *Almagest* has clarified the natural-philosophical commitments that underpin the work, especially as they are presented in the first book, and explored how they can be understood in relation both to his other works and to his broader intellectual milieu.[10] These studies have not, however, examined in detail Ptolemy's defense in *Alm.* 9.2 of his potentially controversial methods, either how it may draw on and reinforce the philosophical apparatus presented in the first book, or how it may rather contribute something new. In this paper, I attempt to remedy this omission by offering a close reading of the philosophical *apologia* of *Alm.* 9.2. My goal is not to present or revise a conception of Ptolemy's philosophy of science as such, but instead to demonstrate how the *Almagest* furnishes a self-contained and coherent philosophical system supporting its astronomical method. I begin (Section 2) by setting out the text of Ptolemy's defense (= i.2. 211.21-212.23). I then frame my reading of the passage in terms of two central observations (Sections 3-4). First (Section 3), I contextualize the problem that prompts the defense and, while sketching its philosophical underpinnings, show how Ptolemy's justification of the equant ultimately depends on the hierarchy of sciences that he adopts in the first book of the *Almagest*. This realization entails a greater appreciation of the importance of Ptolemy's philosophical position for his astronomical endeavors.[11] The second observation (Section 4) concerns Ptolemy's statements on the astronomers' acquisition of the 'first principles' (πρῶται ἀρχαί). I detect in his remarks a possible appeal to Aristotelian doctrine that has not, so far as I am aware, been noticed. Such an appeal would, from an ancient perspective, further strengthen the epistemic force of Ptolemy's defense of the equant and, from a modern one, enrich our understanding of his philosophical resources and method.

10 See Taub 1993, 19-37 and *passim*; Bowen 2007: 349-55; Feke 2009: 17-67 and *passim*; Bernard 2010; Feke and Jones 2010, esp. 202-5; Feke 2012; 2014, esp. 267-70.

11 Bowen 1994, 141 raises the possibility that *Alm.* 1.1 does not represent Ptolemy's views in final form, or, indeed, even his considered opinions; consequently, Ptolemy would be free to discard this—apparently lightly spun—doctrine later, once it had served its purpose of drawing readers into the 'alien conceptual framework' of the *Almagest*. This could be true, but it seems to me that we should resort to such a hypothesis only if all integrating readings fail: the thrust of this paper will be that they do not (and see also the work of Feke and Bernard quoted in the previous note).

2 Ptolemy's Defense of the Equant: Text and Translation of *Alm.* 9.2

It will be useful as a preliminary measure to reproduce in full the crucial passage in which Ptolemy offers his defense of the unorthodox astronomical devices—the equant chief among them—that he will employ in the following books. This 'passage' is in fact one long sentence, and a *prima facie* indication of its importance is its careful language and symmetry. I reproduce it below, breaking it up into more manageable divisions (i.2. 211.21-212.23, tr. modified from Toomer):[12]

> ταῦτα δ᾽ εἴπομεν οὐκ ἐνδείξεως ἕνεκεν, ἀλλ᾽ ὅπως, ἐὰν ὑπ᾽ αὐτοῦ τοῦ πράγματος ἀναγκαζώμεθά που ἤτοι καταχρήσασθαί τινι παρὰ τὸν λόγον, ὡς ὅταν φέρ᾽ εἰπεῖν (1) ὡς ἐπὶ ψιλῶν τῶν ἐν ταῖς σφαίραις αὐτῶν γραφομένων ὑπὸ τῆς κινήσεως κύκλων καὶ ὡς κατὰ τὸ αὐτὸ ἐπίπεδον ὄντων τῷ διὰ μέσων τῶν ζῳδίων διὰ τὸ εὐπαρακολούθητον τὰς ἀποδείξεις ποιώμεθα, ἢ (2) ὑποτίθεσθαί τινα πρῶτα μὴ ἀπὸ φαινομένης ἀρχῆς, ἀλλὰ κατὰ τὴν συνεχῆ διάπειραν καὶ ἐφαρμογὴν εἰληφότα τὴν κατάληψιν, ἢ (3) μὴ ἐπὶ πάντων τὸν αὐτὸν καὶ ἀπαράλλακτον τρόπον τῆς κινήσεως ἢ τῆς ἐγκλίσεως τῶν κύκλων ὑποτίθεσθαι, συγχωρῶμεν εἰδότες, ὅτι (1') οὔτε τὸ καταχρήσασθαί τινι τῶν τοιούτων, ἐφ᾽ ὅσον ουδεμία παρὰ τοῦτο μέλλει παρακολουθεῖν ἀξιόλογος διαφορά, βλάψει τι τὸ προκείμενον, (2') οὔτε τὰ ἀναποδείκτως ὑποτιθέμενα, ἐὰν ἅπαξ σύμφωνα τοῖς φαινομένοις καταλαμβάνηται, χωρὶς ὁδοῦ τινος καὶ ἐπιστάσεως εὑρῆσθαι δύναται, κἂν δυσέκθετος ᾖ ὁ τρόπος αὐτῶν τῆς καταλήψεως, ἐπειδὴ καὶ καθόλου τῶν πρώτων ἀρχῶν ἢ οὐδὲν ἢ δυσερμήνευτον φύσει τὸ αἴτιον, (3') οὔτε τὸ διενεγκεῖν που τὸν τρόπον τῆς ὑποθέσεως τῶν κύκλων θαυμαστὸν ἂν καὶ ἄλογον εἰκότως τις ἡγοῖτο καὶ τῶν περὶ αὐτοὺς τοὺς ἀστέρας φαινομένων ἀνομοίων καταλαμβανομένων, ὅταν γε μετὰ τοῦ κατὰ πάντων ἁπλῶς τὴν ὁμαλὴν καὶ ἐγκύκλιον κίνησιν διασῴζεσθαι καὶ τῶν φαινομένων ἕκαστα κατὰ τὸ κυριώτερον καὶ καθολικώτερον τῆς τῶν ὑποθέσεων ὁμοιότητος ἀποδεικνύηται.

> The point of the above remarks [concerning the inability of Hipparchus and others to account for the anomalies of the five planets, etc.] was not to boast.[13] Rather, if we are at any point compelled by the matter itself to use something that is against theory, as when, for instance, we (1) carry

12 My division of the passage follows a suggestion of Mark Schiefsky. When I refer to any part of it in the remainder of the paper, I will refer it by the section labels here, (1)-(3) and (1')-(3').

13 In the foregoing passage Ptolemy discussed the approach of Hipparchus to the movements of the sun and the moon, but said that the planets' 'continuously compounded'

> out proofs concerning the circles traced out in the planetary spheres by the movement, assuming that these circles are bare and lie in the plane of the ecliptic to make the proof easier; or (2) make some basic hypotheses apprehended not on the basis of an evident principle, but through continuous trial and adjustment; or (3) hypothesize a type of motion or inclination of the circles which is not the same and identical for all planets; we may accede, since we know that (1') the use of such things will not harm our purpose at all, insofar as no significant error will follow from it; and [since we also know] that (2') things hypothesized without proof, provided only that they are found to be in agreement with the phenomena, cannot be discovered without some method and attention, even if the method of grasping them is hard to set out, since also in general there is either no cause of first principles or one that is difficult to describe in nature; and [we know] that (3') one could not plausibly think that a certain difference in the hypothesis of the circles is wondrous and contrary to reason, since the phenomena of the stars themselves are also themselves grasped as different—[one could not think as much] when, at least, with uniform, circular motion being preserved in absolutely all cases, each of the phenomena is also demonstrated according to something that is more authoritative and general than similarity of hypotheses [sc. for all planets].

We will have recourse to this passage in the following sections, but it may simply be noted for now that (1)-(3) articulate Ptolemy's theoretical departures and (1')-(3') provide the justifications for those departures, although they are at times couched in obscure language.

3 The ἀρχαί, the Equant, and Ptolemy's Hierarchy of the Sciences

It is reasonable to begin with the question: What is Ptolemy worried about? Or, in slightly different terms: What is the problem that he anticipates? At the very outset of the passage, he identifies the possible accusation against him, namely, that he may be found 'employing something that is against theory' (καταχρήσασθαί τινι παρὰ τὸν λόγον). The indefinite τινί is to be construed as a sort of catchall for the theoretical novelties enumerated in (1)-(3), but (as the reader will later realize) must refer especially to the equant, the 'paradigm

anomalies (i.2. 209. 3-4, μεμιγμένας ... διὰ παντὸς ἀμφοτέρας), i.e. zodiacal and synodic, defeated him.

case' of such novelties.[14] It is not immediately clear, however, what it means for Ptolemy to face a charge that his method is παρὰ τὸν λόγον. The phrase literally means 'against theory', but not 'against theory' or 'against reason' in an unqualified sense; for that, Ptolemy would use ἄλογος, which he employs in that sense later in the passage. παρὰ τὸν λόγον will mean rather 'against [sc. *some*] theory', but it is natural to ask *whose* or *what* λόγος is it against? If we bracket the 'who' question (which may be unanswerable at our remove),[15] an attentive reading of the passage suggests an answer to the nature of the λόγος that Ptolemy allegedly violates. The language and theoretical concerns of the *apologia* indicate that the problem of the equant is first and foremost a problem with principles (ἀρχαί) and, to a lesser extent, with hypotheses (ὑποθέσεις).

Consider in this regard the programmatic lemma (2), in which Ptolemy says that he 'makes some basic hypotheses apprehended not on the basis of an evident principle' (ὑποτίθεσθαί τινα πρῶτα μὴ ἀπὸ φαινομένης ἀρχῆς … εἰληφότα τὴν κατάληψιν). As becomes evident for those reading further in the *Almagest*, the equant must be one of the τινὰ πρῶτα that Ptolemy is hypothesizing; what is important here and requires further comment is the suggestion of apprehending such τινὰ πρῶτα (sc. πρῶται ὑποθέσεις) on the basis of an ἀρχή. By casting the fact that he has not derived the equant from an ἀρχή as a possible reproach, Ptolemy implicitly admits that the astronomer *should* in general do so, or at least that doing so would conform to theoretical norms of astronomy. This admission both acknowledges the standard function of ἀρχαί in ancient scientific and philosophical contexts and largely agrees with the method on display in the *Almagest*. Generally speaking, ἀρχαί are the true and known principles that *a priori* set the premises and constraints for scientific research. This is indeed what makes them ἀρχαί, 'starting points'.[16] *Qua* first principles, they are self-evident or else have acquired conviction in some way that guarantees their epistemic authority. The astronomer's investigations proceed on the solid foundation that they provide, as he posits additional ὑποθέσεις which,

14 As Toomer (1984 *ad loc.*) aptly puts it.

15 It could be the Stoics (see Bowen 2007, 349-54; Wolff 1988, 497-502; Mueller 2004), but the target may be any group adopting a basically Aristotelian view of the hierarchy of sciences (see Feke 2009, 23).

16 A traditional early source is e.g. Aristotle, *APo.* 71b19-72b4, which is not to say that Ptolemy employed Aristotle's theory of demonstrative knowledge. Yet he may be responding to it: see below Section 4.

in conjunction with the ἀρχαί, establish scientific 'demonstrations' (ἀποδείξεις) accounting for the phenomena.[17]

There is another detail in (2) that is germane: Ptolemy's description of the ἀρχή as φαινομένη, 'evident' or 'manifest'. The description has a double purpose here. First, it recalls the epistemic status of the ἀρχαί by reminding us that they ought to be such as to be apparent to those who are in engaged in the field: as the starting points for further inquiry, they should be grasped (καταλαμβάνεσθαι) and, owing to their transparency, also accepted. The second purpose is to motivate the reader to reflect on the specific ἀρχαί of the astronomer's work, and to investigate which *one* of them could be properly called φαινομένη. In this respect it is important that we find the singular φαινομένης ἀρχῆς, rather than plural φαινομένων ἀρχῶν. The number of starting points that could claim the status of being self-evident is small, and in fact Ptolemy routinely brings up only one principle in the *Almagest* that is described as a *sine qua non* for the astronomer: the supposition that the heavenly bodies move in 'uniform and circular motions' (ὁμαλαὶ καὶ ἐγκύκλιοι κινήσεις).

This 'principle of uniform circular motion' (as we will call it), familiar to students of ancient astronomy, certainly did not originate with Ptolemy; nor can we be certain about its exact form when he received it. Moreover, it should be observed that he never *explicitly* calls the requirement of preserving uniform circular motion an ἀρχή. That is, we do not find in the text of the *Almagest* the words (e.g.) ἡ ἀρχὴ ἡ τῶν ὁμαλῶν καὶ ἐγκυκλίων κινήσεων. Nevertheless, Ptolemy's remarks show that he conceives of it as functioning as an ἀρχή and, what is more, that he thinks of it as the defining principle or constraint of his project. Three quotations, each in a different but important context, will suffice to establish the validity of these claims:

17 This description of Ptolemy's method is supported by the development of the *Almagest*, whereby Ptolemy argues for a number of privileged hypotheses in *Alm.* 1.2-7 that are treated as necessary preliminaries for what he calls the 'demonstrations' (ἀποδείξεις) that will follow: see i.1. 26.6-8 and 30.19-22, and Taub 1993, 39-45. But here is an interesting problem: although Ptolemy treats the hypotheses of *Alm.* 1.2-7 as ἀρχαί, he does not call them by that name. This *may* be because he takes an ἀρχή to be nothing more than a ὑποθέσις that has a special status, i.e. that of being epistemically beyond reproach; and, further, if he really believes what he says at (2'), i.e. that the cause of ἀρχαί either does not exist or is prohibitively difficult to explain, then he would be justified in avoiding the term in reference even to the hypotheses about which he is fairly confident. (Add that the hypotheses of *Alm.* 1.2-7 are achieved at least in part by arguments, and for that reason may not themselves be considered truly first principles.) Regardless of that problem, there can be no doubt in light of *Alm.* 9.2 that Ptolemy knows what role the ἀρχαί should or would play.

> With regard to the determination of the positions of the sun and the other [heavenly bodies] for any given time … we think that the mathematician's task and goal ought to be to show all the heavenly phenomena being produced by uniform circular motions (δι᾽ ὁμαλῶν καὶ ἐγκυκλίων κινήσεων ἀποτελούμενα) (i.1. 208.15-21, tr. modified from Toomer).

> First we must make the general point that the rearward displacements of the planets with respect to the heavens are, in every case, just like the motion of the universe in advance, by nature uniform and circular (ὁμαλαί … εἰσιν πᾶσαι καὶ ἐγκύκλιοι τῇ φύσει) … The apparent irregularity in their motions is the result of the position and order of those circles in the sphere of each by means of which they carry out their movements, and in reality there is nothing alien to their eternal nature in the disorder which the phenomena are supposed to exhibit (i.1. 216.1-16 , tr modified from Toomer).

> Now it is our purpose to demonstrate for the five planets, just as we did for the sun and moon, that all their apparent anomalies can be produced by uniform circular motions (δι᾽ ὁμαλῶν καὶ ἐγκυκλίων κινήσεων ἀποτελουμένας), since these are proper to the nature (τῇ φύσει) of divine beings, while disorder and non-uniformity are alien [to such beings]. Then it is right that we should think success in such a purpose a great thing, and truly the proper end of the mathematical part of theoretical philosophy (i.2. 208.4-11, tr. modified from Toomer).

It will be noted that Ptolemy does not undertake in these passages to demonstrate or in any other manner to prove the proposition that the planets move only in uniform circular motions. Rather, he takes it for granted that it is true, appealing to the nature (φύσις) of the heavenly bodies, and sets it as his goal to demonstrate how their anomalous or irregular motions can be explained *just on the basis of such an assumption*, i.e. that in 'absolutely all cases' (i.1. 216.9, ἐπὶ πάντων ἁπλῶς) the motion is ὁμαλός and ἐγκυκλίος. But this sort of assumption is precisely what a first principle (ἀρχή) is: a starting point, taken to be true—by nature (τῇ φύσει) in this case—that furnishes the ground for scientific research and demonstration. It is therefore not inappropriate to speak of his stipulation for uniformity and circularity in planetary motion as an ἀρχή, whether or not Ptolemy himself ever does so.

Taking into view the evidence elsewhere in the *Almagest*, the best, and probably only, candidate for the φαινομένη ἀρχή that Ptolemy admits to neglecting in (2) is therefore the principle of uniform circular motion. But supposing that the generalizing τινὰ πρῶτα refers above all to the equant, and that the ἀρχή it fails to conform to is in fact that of uniform circular motion, *why*, we may ask, does the equant so fail in this respect? This question goes to the heart of the principle's significance for Ptolemy's construction of the planetary models, and raises in particular the question what it means for planetary motion to be 'uniform', and how 'uniform' it must be.[18] A preliminary answer can be offered by Ptolemy's mathematical definition of uniform circular motion at *Alm.* 3.3 (i.1. 216.7-11, tr. modified from Toomer):

> If we imagine the bodies or their circles being carried around by straight lines, in absolutely every case the straight line in question describes equal angles at the center of its revolution in equal times.

As Ptolemy goes on to argue (i.1. 216.16-217.6), this principle can be preserved in eccentric and epicyclic models, albeit with a slight twist, if one allows the motion to be uniform with respect either to the eccentric or, in the case of an epicycle, to the center of the deferent. But the equant is apparently a different matter. Whereas Ptolemy does not seem worried about objections to the introduction of the eccentric or epicycle *per se*, his defensive admission in *Alm.* 9.2 regarding the τινὰ πρῶτα, among them the equant, shows where the problem lies. I submit that this defensiveness arises because he foresees the objection that the equant does not preserve uniformity in the same way as the eccentric and epicycle: that is, it is not derived from the φαινομένη ἀρχή of uniformity. In particular, for orthodox eccentric and epicyclic theories, the center of revolution remains in an important sense the center from which the uniformity of motion is observed (Figures 1 and 2 above); but, because the equant shifts the point of uniformity away from the center of revolution, it fails to preserve the principle of uniform circular motion as Ptolemy describes it in *Alm.* 3.3, even if it does not do away with uniformity *tout court* (Figure 3).[19] This discrepancy, we are now in a position to remark, is what is likely meant by παρὰ τὸν λόγον: Ptolemy goes 'against the λόγος' that he himself prescribed, and, as is probable, against the same λόγος to which his contemporaries adhered. It may be telling at any rate that Ibn al-Haytham and Copernicus found the equant intolerable for just this reason.[20]

Adopting a broader perspective here will allow us to weigh up the potential difficulties posed by the equant's departure from theory (λόγος), the seriousness of which depends to a great extent on the authority that one attributes to the principle of uniform circular motion. For those who take the principle in its strict sense to be immutable and indispensable, Ptolemy's equant cannot

18 See Neugebauer 1975, 55-7; Jones 2004, 375-6.

19 On the uniformity preserved by eccentric and epicyclic models, see Pedersen 2011, 134-7.

20 See n. 6 above.

be justified. But what if the ἀρχή is not immutable? What if in its strict form it is taken only as a *regulative* principle, one that will need to be adjusted in light of the research into φαινόμενα that it guides? In a word, I suggest that this is exactly how Ptolemy means to justify the equant: while the equant cannot be 'apprehended' (εἰληφότα τὴν κατάληψιν, (2)) or otherwise derived from the principle of uniform circular motion *stricto sensu*, that does not mean that it is epistemically unjustified in an absolute sense (i.e. ἄλογον (3')).

On the contrary, Ptolemy maintains that he did not introduce the equant 'without some method and attention' (χωρὶς ὁδοῦ τινος καὶ ἐπιστάσεως (2')). Recent scholarship has emphasized that this method is an empirical and mathematical one,[21] but there is no need to look beyond our passage for evidence of that fact: Ptolemy himself admits to replacing the φαινομένη ἀρχή with 'continuous trial and adjustment'[22] (τὴν συνεχῆ διάπειραν καὶ ἐφαρμογήν (2)).[23] However brief this notice,[24] what it amounts to in context is the adoption of a substantive philosophical position that prioritizes the power of observation and mathematical experiment, implicit in the language of διάπειρα and ἐφαρμογή, over the self-evidence (φαινομένη) of the ἀρχή to reveal the nature of the planets. Indeed, Ptolemy clinches this point when he says that it is agreement with φαινόμενα (ἐὰν ἅπαξ σύμφωνα τοῖς φαινομένοις καταλαμβάνηται (2')), not with any particular ἀρχή, that is the decisive factor in recommending a (i.e. *his*) scientific method. He goes on to reinforce this position with the claim that the cause (αἴτιον) of the πρῶται ἀρχαί is either nonexistent or difficult to explain (ἢ οὐδὲν ἢ δυσερμήνευτον (2')), and that the way towards their discovery is hard to set out (δυσέκθετος). This admission undercuts the authority of the φαινομένη ἀρχή and suggests the possibility of alternative routes to justifying the first principles, indeed, of alternative first principles altogether. By requiring in particular that the astronomer's results be held to account against the φαινόμενα, Ptolemy moves to establish the source of the ἀρχαί, at least in part, within the astronomer's essential purview of observation and mathematics:

21 See e.g. Swerdlow 2004*a*, 249-50 and *passim*; 2004*b*, 140 and *passim*; Bowen 2007, 352-4.

22 For further thoughts on ἐφαρμογή as 'adjustment', see n. 25 below.

23 On possible mathematical routes to the equant, see Neugebauer 1975, 152-6; Evans 1984; Jones 2004; Swerdlow 2004*a*; Duke 2005*a*; Pedersen 2011, 273-87; Gamini and Hamedani 2013. For comparative evidence, see Van der Waerden 1961; Duke 2005*b*.

24 As discussed below, it is indeed too brief to justify the position it reflects, which is set forth in greater detail in Book One.

agreement with the φαινόμενα must ultimately validate the hypotheses the astronomer provides.[25]

Thus Ptolemy adopts in *Alm.* 9.2 the position that observational and mathematical exigency will allow the astronomer to modify or prescribe his own ἀρχαί in response to his research. If true, the equant will be a paradigm case of such an operation. It is a surprising, but mathematically satisfying, departure from the expected sense of the principle of uniform motion: it conforms not to the evident ἀρχή, but to the φαινόμενα. The reader will observe, however, that the empirical method of trial and error that Ptolemy advocates here is merely asserted—not proven—, and it is a matter of some doubt whether he could have expected all astronomers and natural philosophers to agree with it. For there was a substantial tradition from at least the time of Aristotle[26] which held that astronomy could not prescribe or modify its own principles of investigation (ἀρχαί). More conservative astronomers or natural philosophers could, on the basis of this tradition, make a reasoned case for the illegitimacy of the equant. In order to appreciate what kind of problem Ptolemy may have faced from this quarter, we will briefly sketch the most important points of the Aristotelian position.[27]

It is well known that Aristotle distinguishes among three branches of theoretical sciences (*Metaph.* 1026a18-19, φιλοσοφίαι θεωρητικαί): theology, physical theory and mathematics.[28] I will say no more about theology, for it is not germane to our purposes here. Physical theory studies moved objects, both

25 This point can be illustrated by some passages in which Ptolemy appeals to the φαινόμενα in order to confirm his argument. Especially interesting are the concluding remarks of i.2. 269.3-5 (on the motion of the apogees): ἔκ τε δὴ τούτων καὶ ἐκ τῆς τῶν περὶ τοὺς ἄλλους ἀστέρας φαινομένων κατὰ μέρος ἐφαρμογῆς … εὑρίσκομεν κ.τ.λ. We see the recurrence here not only of φαινόμενα but also ἐφαρμογή: the appearance of the latter word at *Alm.* 9.2 is well translated 'adjustment', but the present passage is important in that it shows that this adjustment is really only a check on the 'fit' of a demonstration (as Toomer translates here) with respect to the phenomena. One should also consider, for a different reason, i.1. 26.6-12, where Ptolemy says that the physical ὑποθέσεις of *Alm.* 1.2-7 'will be completely confirmed and further proven by agreement with the theories of the phenomena which we shall demonstrate in the following sections' (βεβαιωθησομένας τε καὶ ἐπιμαρτυρηθησομένας τέλεον ἐξ αὐτῆς τῆς τῶν ἀκολούθως καὶ ἐφεξῆς ἀποδειχθησομένων πρὸς τὰ φαινόμενα συμφωνίας). The point is remarkably consonant with that that of (2'): compare esp. σύμφωνα τοῖς φαινομένοις with πρὸς τὰ φαινόμενα συμφωνίας.

26 But not necessarily confined to the Peripatetics: see n. 15 above.

27 Cf. Mueller 2004; Bowen 2013, 37-57.

28 For further characterization of each of these branches, cf. *Metaph.* 1025b3-1026a32 and *Ph.* 193b22-194a18.

sublunary and superlunary, and investigates their essential properties.[29] In contrast, mathematics tends to proceed abstractly, and its true domain is number and other mathematical properties not *qua* instantiated but on their own terms.[30] Yet Aristotle allows that there exist certain branches of mathematics that are 'more physical' (*Ph.* 194a 7-8, τὰ φυσικώτερα τῶν μαθημάτων) because they study instantiated mathematical properties. These include optics, harmonics and astronomy.[31] The 'mixed' nature of such branches puts them in a position of unique interdependence. Mathematics *ex hypothesi* has no claim to know what is essentially the case about the *realia* it studies, either in general or in particular;[32] it can proceed only on the basis of the knowledge that it obtains from physical theory. It is crucial, then, that Aristotle defines astronomy as a branch of mathematics, for it follows from this that astronomy itself has little or no authority to investigate the natural properties of the heavenly bodies: the astronomer is compelled to rely on the physical theorist in order to obtain the first principles (ἀρχαί) that are relevant for his study of the heavenly bodies, such as the kind of movement that they undergo.[33] A passage from Simplicius, reporting Posidonius via Geminus, may be instructively quoted

29 See *Metaph.* 1026a14 for the claim that physics studies moved bodies (οὐκ ἀκίνητα). By 'essential properties' I mean that the task of the physical theorist according to Aristotle is to determine what belongs to their φύσεις both essentially and necessarily (on this last distinction, see van Fraassen 1980, 31-2 and *passim*). For the task of physical theory in general, see *Ph.* 192b22-194b15; cf. *De An.* 403a24-b19.

30 See e.g. *Ph.* 193b22: οὐδὲ τὰ συμβεβηκότα θεωρεῖ [sc. ἡ μαθηματική] ᾗ τοιούτοις οὖσι συμβέβηκεν ('nor does mathematics consider accidentals as they inhere in beings'). Also important is *Metaph.* 1061a28-b3.

31 See also *Metaph.* 1026a25-7. Astronomy is again implicitly defined as a branch of mathematics at 1073b4-5, where Aristotle says it is the 'closest of the mathematical branches of knowledge to [physical] theory' (ἐκ τῆς οἰκειοτάτης φιλοσοφίᾳ τῶν μαθηματικῶν ἐπιστημῶν … ἐκ τῆς ἀστρολογίας). See also Mueller 2006.

32 Which is not to say that mathematics has no role at all in determining the φύσις (nature) of these objects, since φύσις comprises not just the matter (ὕλη) of an object but also its form (εἶδος), some aspects of which mathematics may investigate: cf. *Ph.* 193b22-194a12. In the *Physics* passage, Aristotle is quick to add, however, that the *kind* of form that we are concerned with in physical investigation is not the form *qua* mathematical, but only as physically instantiated: so *Ph.* 194a12-15: ἐπεὶ δ᾽ ἡ φύσις διχῶς, τό τε εἶδος καὶ ἡ ὕλη, ὡς ἂν εἰ περὶ σιμότητος σκοποῖμεν τί ἐστιν, οὕτω θεωρητέον· ὥστ᾽ οὔτ᾽ ἄνευ ὕλης τὰ τοιαῦτα οὔτε κατὰ τὴν ὕλην ('and since nature is twofold, form and matter, we must investigate it just as if we were to consider what snubness [of the nose] is: that is, we [must investigate] those sorts of things neither without matter nor [only] in terms of matter').

33 A good illustration of this method in practice can be found in *Metaph.* 1073a3 *ff.*, where Aristotle first uses arguments from physical theory to determine the essential properties

to demonstrate the relationship between astronomy and physics in this vein (*in Phys.* ix. 292.23-29 Diels = Posidonius F18.42-9 Edelstein and Kidd):[34]

> For it is far outside the astronomer's purview (οὐκ ἔστιν ἀστρολόγου) to know what is naturally at rest and what sort of things are in motion; he rather proposes hypotheses that some things remain still and that others move, and searches for the hypotheses with which the heavenly phenomena will agree (τίσιν ὑποθέσεσιν ἀκολουθήσει τὰ κατὰ τὸν οὐρανὸν φαινόμενα). But he must take his first principles from the physicist, that the motions of the stars are simple and uniform and ordered (ληπτέον δὲ αὐτῷ ἀρχὰς παρὰ τοῦ φυσικοῦ,[35] ἁπλᾶς εἶναι καὶ ὁμαλὰς καὶ τεταγμένας κινήσεις τῶν ἄστρων), by means of which he will demonstrate the circular motion of all the stars that revolve along either the parallel or the oblique circles.

The value of Simplicius' passage for our purposes, regardless of its evidentiary quality for the beliefs of Ptolemy's contemporaries, is to show the durability and coherence of the hierarchy of sciences that originated with Aristotle. The sort of reasoning sketched above and reflected in Simplicius poses a real threat to the equant: against Ptolemy's assertion of empirical and mathematical considerations that would justify it, we must balance a fairly ancient and coherent tradition according to which the astronomer's exploration of planetary movement is subservient to the first principles (ἀρχαί) obtained from physical theory.

In fact, Ptolemy seems to be fully aware of the challenge that the equant faces along Aristotelian lines, and he does not leave it to the assertions of *Alm.* 9.2 alone to underwrite his revision of the principle of uniform circular motion—far from it. Those assertions rather point back to a fuller and more sophisticated explanation of the astronomer's ability to modify the ἀρχαί:

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of heavenly bodies and only then engages astronomy to add *a posteriori* information about the number of reactive spheres that carry them.

34 My purpose in quoting the passage is not to suggest that Ptolemy was responding directly to Geminus, but only to show the durability and vigor of Aristotle's thesis. On the passage itself, see Bowen 2007: 344; 2013: 40-50.

35 With Posidonius' language it is worth comparing Aristotle's statements in the *De respiratione* on the relationship between the doctor (ἰατρός) and natural scientist (φυσικός). There, Aristotle says that 'all of the clever and inquisitive doctors say something about nature, and think it appropriate to take their principles from there' (480b26-8: τῶν τε γὰρ ἰατρῶν ὅσοι κομψοὶ καὶ περίεργοι λέγουσί τι περὶ φύσεως καὶ τὰς ἀρχὰς ἐκεῖθεν ἀξιοῦσι λαμβάνειν). Cf. also *De sensu* 436a17-436b1.

I mean here the modification of the hierarchy of sciences in the first book of the *Almagest*, where Ptolemy appropriates Aristotle's hierarchy but reorganizes it, elevating mathematics—and along with it astronomy—to the highest position among the theoretical sciences. (I do not know if it is necessary to point this out, but Ptolemy's awareness of Aristotle's hierarchy surely suggests that he was aware of the challenge to the equant that it might pose: that is his reason for revising it.) This reorganization has received much commentary and elucidation in recent years,[36] and I shall offer only a few salient observations.

Ptolemy claims that mathematics alone has the right to be called 'sure apprehension' (i.1. 6.13, κατάληψιν ἐπιστημονικήν). He argues that physics can attain to no more than 'guesswork' or conjecture (i.1. 6.12, εἰκασίαν), because it must wrestle with the 'unstable and unclear nature of matter' (i.1. 6.15, τὸ ὕλης ἄστατον καὶ ἄδηλον); theology also falls short of knowledge, focusing as it does on an opposite extreme, the realm of the 'invisible and ungraspable' (i.1. 6.14, τὸ παντελῶς ἀφανές … καὶ ἀνεπίληπτον). Mathematics traverses the mean between these studies, attending to what is capable of abstraction and certain exactness in objects (number), yet avoiding the prohibitively distant objects of theology. For this reason, mathematics alone provides 'sure and unshakeable knowledge' (i.1. 6.18-19, βεβαίαν καὶ ἀμετάπιστον … τὴν εἴδησιν). The important consequence of all of this, as Ptolemy makes clear, is that astronomy need not look exclusively to physics for knowledge of the natural bodies. Rather, it may itself contribute to their discovery (i.1. 7.10-17, tr. Toomer):

> As for physics, mathematics can make a significant contribution. For almost every peculiar attribute of material nature becomes apparent from the peculiarities of its motion from place to place. [Thus one can distinguish] the corruptible from the incorruptible by [whether it undergoes] motion in a straight line or in a circle, and heavy from light, and passive from active, by [whether it moves] towards the center or away from the center.

In this passage, Ptolemy provides a list of essential (i.e. natural) properties of the heavenly bodies which were traditionally acquired by way of physical theory and taken as the premises for astronomical investigation, i.e. as ἀρχαί. He points out, however, that all of these ἀρχαί could as easily be ascertained by the mathematician, whose observations regarding the mathematical properties of

36 See most fully Feke 2009, 17-67; also Taub 1993, 19-37; Bowen 2007, 349-55; Feke 2014, 267-70.

objects attest also to their nature. To generalize, the first principles can flow not only from physics to mathematics, but also the other way around.

The reorganization of sciences in the first book of the *Almagest* proves from a philosophical perspective to be critically important to the solution of the planetary anomalies in Book Nine and following. By furnishing astronomy with the ability to provide its own first principles, Ptolemy also authorizes the introduction of the equant, which is based precisely on the fact that the φαινόμενα may demand modification of the ἀρχαί in response to experiment and observation. The philosophical doctrine of the first book of the *Almagest* thus comes to fruition in the ninth book: I do not mean in the *apologia* of 9.2, but in the actual arguments that follow. The *apologia* itself, however, plays the not insignificant function of *aide-mémoire* and philosophical prop, recalling for the reader the significance of the points set out in the first book and indicating how Ptolemy will apply them.[37]

Taking into view Ptolemy's revision of the hierarchy of sciences, the reader of the *Almagest* will see that there is no insuperable contradiction in Ptolemy's claim that with the equant he has 'preserved uniform and circular motion in absolutely all cases' (κατὰ πάντων ἁπλῶς τὴν ὁμαλὴν καὶ ἐγκύκλιον κίνησιν διασῴζεσθαι (3')), despite the fact that this statement seems to be belied by the definition of uniform motion offered at *Alm.* 3.3. For it is no longer strictly an immutable ἀρχή that is the decisive factor, but rather 'something that is more authoritative and general than similarity of hypotheses' (τὸ κυρίωτερον καὶ καθολικώτερον τῆς τῶν ὑποθέσεων ὁμοιότητος, (3')). This somewhat mysterious closing statement can be elucidated as follows: we should understand the 'similarity of the hypotheses' to refer to a feature that Ptolemy takes to be contained implicitly in the principle of uniform circular motion, namely that the principle must be applied in the same way (ὁμοιῶς) to all of the heavenly bodies. Here is the most economical reasoning for this 'similarity': since (premise) all the heavenly bodies are made of the same substance (*aether*), and (premise) uniform circular motion holds of this substance in general, not any particular instance of it, (inference) uniform circular motion will hold in the same way for all bodies composed of the substance. But further reflection shows that these premises are insufficient to support the inference: we could only obtain it if we were to assume some other premise, for example, that *nothing but* the substance influences the motion of the heavenly bodies.

37 Cf. also *Alm.* 13.2, which bookends the arguments that follow the *apologia* of *Alm.* 9.2 and stresses again the primacy of the φαινόμενα in determining the properties of the heavenly bodies.

As an alternative to a strong application of the 'similarity of the hypotheses' that incorporates this additional premise, Ptolemy offers the riddling τὸ κυρίωτερον καὶ καθολικώτερον. While we cannot be sure what precisely he had in mind by the phrase, I submit that by the time the reader has arrived at this point the *apologia* of Book Nine, he will have more than enough evidence to warrant a good guess: Ptolemy will be referring obliquely to the core of his own astronomical method, mathematical experiment in response to observation of the φαινόμενα, which, owing to his elevation of the status of mathematics, is in a position to trump the epistemic commitment to any particular physical principle. Ptolemy's departure from the 'simple' understanding of the principle of uniform circular motion thus ultimately serves an end that is higher and more authoritative (κυρίωτερον καὶ καθολικώτερον). In the final analysis, empirical considerations will falsify the strongest application of the similarity of hypotheses in favor of a cosmological model that reconciles observation to mathematics.

4 Aristotle and the Search for ἀρχαί

In recent years, the role of Aristotle's philosophy in the *Almagest* has been de-emphasized in important respects. This is an appropriate corrective to the enthusiasm of earlier commentators to find a Peripatetic basis for Ptolemy's philosophy.[38] But, as scholars have shown for Book One at least, Ptolemy is more than capable of employing the language and substance of Aristotelian doctrine in order to fend off contemporary critics and frame his own philosophical commitments.[39] In this section, I will argue that we may be able to detect another appeal to Aristotelian doctrine in the *apologia* of *Alm.* 9.2. As we have already seen, Ptolemy complains in (2') that the cause (αἴτιον) of the first principles (πρῶται ἀρχαί) is either nonexistent or difficult to explain (ἢ οὐδὲν ἢ δυσερμήνευτον), and that it is hard to set out (δυσέκθετος) how they may be found. For this reason, he says, he may be excused for hypothesizing some things without proof (τὰ ἀναποδείκτως ὑποτιθέμενα), because such a procedure will be justified once their agreement with the phenomena has been established. With these remarks, Ptolemy may seem *prima facie* to be doing no more than excusing the introduction of the equant by pointing to the difficulty of his task. As I argue, however, it is in Ptolemy's talk of the πρῶται ἀρχαί, and

38 Especially Boll 1894. For corrective assessments, see the work of Feke, Taub, and Bowen cited n. 10 above.

39 See esp. Feke 2009, ch. 2.

the need to employ an alternate method for attaining them, that an appeal to Aristotelian doctrine can be found.

We begin with some philological observations. Ptolemy's employment of the phrase πρῶται ἀρχαί in the context of the careful language of the *apologia* can be taken as a first indication that he has Aristotle, or an interpreter or exponent of Aristotle, in mind.[40] So far as the written record indicates, this pairing of words, rare in Classical Greek, was used as a unified concept for the first time in Aristotle's writings, where it appears infrequently but always with great significance. The phrase would go on to become quite common among Aristotle's late-Christian-era commentators, who used it both far more frequently and in a greater variety of contexts than had Aristotle himself; and although we do find it occasionally employed without reference to Aristotle, the great majority of its occurrences are found in contexts where it refers directly or indirectly to Aristotelian doctrine.[41] This is the only time that the *iunctura* πρῶται ἀρχαί with the meaning 'first principles' occurs in the Ptolemaic corpus. It seems harder to believe that the words appear by coincidence only here, in the philosophically significant context of *Alm.* 9.2, than that Ptolemy intends a reference to Aristotelian doctrine, even if it is left an open question whether he is thinking immediately of Aristotle himself or of another Peripatetic philosopher.

There are a handful of other possible allusions to Aristotle's language in (2'),[42] but the philological evidence will persuade only so far. In order to make the suggestion of an allusion to Aristotle more plausible, I will argue on the basis of Aristotle's discussion of the πρῶται ἀρχαί that there would be some real attraction to Ptolemy in appealing to his doctrine in defense of the equant. From

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40 It may be that the *hapax* δυσέκθετος comes from an Aristotelian intermediary, but Ptolemy could as easily have coined the term himself.

41 For an example of the rare use of πρῶται ἀρχαί without any special connotation, cf. Plutarch, *De fort. Rom.* 495C; and, in a more philosophical context, but without reference to Aristotle, cf. [Basil], *Enarratio in Isaiam* 5.152.8-9 (*bis*).

42 For example, the word ἀναποδείκτως appears only twice in Ptolemy's writings, in (2') and in the passage immediately preceding (i.2. 212.11). The word is common enough among later writers of all stripes, but the adjective ἀναπόδεικτος appears first in the written records in Aristotle's writings where, among other uses, it is used to describe the indemonstrable primitives (πρῶτα: cf. e.g. *APo.* 71b27). Another interesting but inconclusive suggestion is Ptolemy's pairing of θαυμαστόν … καὶ ἄλογον (only here in his writings): one cannot make much of this, but it is interesting to note that the phrase (specifically θαυμαστὸν καὶ παντελῶς ἄλογον) appears at Aristotle, *Cael.* 269b7 in his very discussion of the 'simple', 'continuous', and 'eternal' motion of the heavenly bodies (μόνην … συνέχη ταύτην τὴν κίνησιν καὶ ἀΐδιον).

the passages in which Aristotle treats the first principles of scientific inquiry, I would like to focus on two here: *Physics* 1.1 (184a10-b14: τὰς ἀρχὰς τὰς πρώτας at 184a13) and *Posterior Analytics* 2.19 (99b20-100b17: τὰς πρώτας ἀρχάς at 99b21). The explicit subject of both these passages, which have been the occasion for much learned commentary, is the character and method of discovery of the πρῶται ἀρχαί. The thrust of Aristotle's conclusions in these passages is germane to Ptolemy's purpose. Moreover—and this point may be more significant than it would at first seem—if Ptolemy were to find a justification within Aristotle's philosophy for his own empirical method and modification of the ἀρχαί, then this fact might silence critics of a Peripatetic leaning or, more widely, any who had taken up Aristotle's hierarchy of sciences and objected on that basis to the equant.

Let us start from *Physics* 1.1. Aristotle observes there that we only say that we know something when we have become acquainted with its 'first causes and first principles' (184a13-14, τὰ αἴτια ... τὰ πρῶτα καὶ τὰς ἀρχὰς τὰς πρώτας). It is, accordingly, the task of one making an inquiry into nature (τῆς περὶ φύσεως ἐπιστήμης) to investigate these principles so as to place scientific demonstration on a solid foundation. Aristotle offers the following advice on how this investigation is to proceed: 'the natural method [for finding principles] is from things more known and clearer to us to things clearer and more known in nature' (184a16-18, πέφυκε δὲ ἐκ τῶν γνωριμωτέρων ἡμῖν ἡ ὁδὸς καὶ σαφεστέρων ἐπὶ τὰ σαφέστερα τῇ φύσει καὶ γωνριμώτερα). As Wieland argued (1975, 128, 132), the significance of Aristotle's recommendation here is to establish that we come to know the principles not through 'pure, immediate intuition' or through 'strict syllogistic reasoning', but through an 'investigation into what presuppositions we have already made if we speak of natural things and events' (i.e. τὰ γνωριμώτερα ἡμῖν καὶ σαφέστερα). Aristotle's insistence upon beginning from those data has the corollary that a principle is judged 'only with respect to what is actually achieved if one attempts to account for [that] state of affairs by introducing it' (132). Inquiry begins from that confusion and moves towards the principles, so that the principles 'stand at the end, not at the beginning of the investigation' (135): far from being given *a priori* or acquired through pure intuition, the πρῶται ἀρχαί are rather the fruit of a searching examination of nature, both inductive and deductive.

It has been suggested that we can also read Aristotle's account of the grasp of first principles in *Posterior Analytics* 2.19 along these lines.[43] In this passage, Aristotle is again concerned with how we come to know the πρῶται ἀρχαί, but here the focus of his solution is on the roles that perception (αἴσθησις) and

43 See Striker 1991 on Irwin 1988.

induction (ἐπαγωγή) play in that process. The details of his account are not pellucid, but his concluding statements capture the essence of his position: 'thus it is clear that it is necessary for us to become acquainted with the first [principles] (τὰ πρῶτα) through induction; for in that way perception, too, instills the universal' (100b3-5, δῆλον δὴ ὅτι ἡμῖν τὰ πρῶτα ἐπαγωγῇ γνωρίζειν ἀναγκαῖον· καὶ γὰρ ἡ αἴσθησις οὕτω τὸ καθόλου ἐμποιεῖ). What is most significant about the inductive account that Aristotle offers in *Posterior Analytics* is the fundamental agreement in methodological outlook that it shares with the *Physics*. Although *APo.* 2.19 remains open to a variety of readings, the basic movement from perception to principles mirrors and indeed seems to develop the epistemological movement of the *Physics* from what is more known and clearer to us (τὰ γνωριμώτερα ἡμῖν καὶ σαφέστερα), which includes the data of sense perception, to what is more known and clearer by nature (τὰ σαφέστερα τῇ φύσει καὶ γωνριμώτερα), which includes first principles.[44] The method in both the *Posterior Analytics* and the *Physics* is cautious: it is not one of positing immutable ἀρχαί at the outset that will constrain our inquiry, but rather one of examining the phenomena so as to end with the ἀρχαί that could provide a satisfactory basis for their explanation.

What is the upshot of Aristotle's method in *Physics* 1.1 and *Posterior Analytics* 2.19 for Ptolemy? It is, I believe, that either or both of these passages—or their reflection in later Peripatetic philosophy—could have suggested to Ptolemy another way of underwriting the modification of the ἀρχαί that the equant represents. As we have seen, Aristotle holds that the first principles are determined above all by the work that they can do in explaining our experience of nature: ἀρχαί that fail to account for what we observe in nature cannot be called ἀρχαί at all, since, even if they are not entirely wrongheaded,[45] they do not adequately explain the phenomena and are *ipso facto* not what we expect of ἀρχαί. The ἀρχαί stand at the end of the investigation, and can only be secured by carefully working through observed phenomena. These conclusions cohere well with Ptolemy's method in general, and would corroborate his insistence that it is a check against the φαινόμενα which determines the validity of one's conclusions (ἐὰν ἅπαξ σύμφωνα τοῖς φαινομένοις καταλαμβάνηται) and by extension the principles that one should adopt. In a fairly straightforward

44 This is not to say that one may easily reconcile the synthetic and inductive method of the *Posterior Analytics* with the analytic method propounded at the beginning of the *Physics*; indeed much ink has been spilt on the question of the relationship between the passages. Their common conclusion in the ἀρχαί cannot be disputed, however, and is the important fact for our purposes.

45 On Aristotle's epistemological optimism, cf. Striker 1991, 492.

way, Aristotle's theorization of the search for the ἀρχαί would thus also be salutary to Ptolemy from the perspective of justifying the equant, for, as we have already noted, the equant can be seen as a paradigm case of the adjustment of an ἀρχή on the basis of empirical research.

Supposing that an appeal to Aristotelian philosophy can be found in (2'), we should also read it into other portions of the *apologia*. We would then be in an even better position to appreciate the significance of Ptolemy's claim that he is relying on 'some method and attention' (2'), and that this method consists in 'continuous trial and adjustment' (2). As Aristotle does, Ptolemy takes up the working potential of the ἀρχή with respect to observation as the most important criterion of its success: we saw that he signals this explicitly with his insistence that his undemonstrated assumptions (τὰ ἀναποδείκτως ὑποτιθέμενα) cannot be criticized 'provided only that they are found to be in agreement with the phenomena' (2'). Moreover, the enigmatic statement at the end of (3') could be read again in light of an appeal to Aristotle. 'Something that is more authoritative and general than similarity of hypotheses' (τὸ κυρίωτερον καὶ καθολικώτερον τῆς τῶν ὑποθέσεων ὁμοιότητος) would be understood as the—basically Aristotelian—experimental and observational method itself that guides the discovery of ἀρχαί, a method which, as Ptolemy might maintain, is superior to a stubborn devotion to any particular ἀρχή, including that of uniform circular motion.

Against the above reasoning it may be objected that compatibility between the doctrines of Ptolemy and Aristotle does not automatically warrant positing a direct intellectual link: affinity need not imply intellectual influence.[46] But while an allusion may not be secured beyond any shadow of doubt, it is clearly more than compatibility that recommends the possibility. To bring the evidence together in conclusion: first, Ptolemy knows and employs Aristotle's philosophy when setting up his philosophical position in the first book of the *Almagest*: using the language and substance of Aristotelian philosophy to ground his method in 9.2 would cohere with his strategy there. Secondly, Ptolemy writes the words πρῶται ἀρχαί in a passage explicitly concerned with scientific method and the status of the ἀρχαί. Readers will need to consider whether it is more likely that the distinctly Aristotelian *iunctura* is coincidental or that an allusion is intended; this consideration should take into account the fact that, *if* one accepts the allusion, it is not stray but refers rather to a doctrine that corroborates the point that Ptolemy is making. And third, the principal opposition to

46 For the same point in the context of medicine, see Schiefsky 2005, 3, 46-7.

the equant is likely to come from an Aristotelian basis (see Section 3 above). The rhetorical value of overturning such Aristotelian critics by appealing to Aristotle's doctrine is obvious: if Ptolemy's critics *were* to attack him on the grounds that he illegitimately used mathematics to modify the physical ἀρχαί, it would be all the better if he could find a means of countering them on the basis of the philosophical system that they suppose him to violate.

5 Conclusion

It must be right, as interpreters have stressed, that Ptolemy's philosophy is to be read in light of his astronomical methods. He was above all a great astronomer and a great scientist, and it would be very surprising if his experience in those fields did not have a decisive influence on his philosophical commitments. But this is not to say that we should discard these commitments as an afterthought, or as somehow irrelevant for his greater ambitions. There is no good reason to believe that Ptolemy himself did not take them seriously, and his care in treating general questions of natural philosophy in the *Almagest* should be taken to indicate that he believed such considerations to inform and complete his explanation of the heavenly phenomena. At the very least, a kind of principle of charity will encourage us to search for integrating readings that take Ptolemy's philosophical avowals at their word and attempt to understand them in the context of his scientific work, as he seems to have wished himself: *Alm.* 9.2 is only one passage among several in the *Almagest* that show Ptolemy anxious for others to understand the philosophical assumptions that underpin his work.

I hope in this paper to have shed further light on Ptolemy's natural philosophy in the *Almagest* and its relationship to his all-important project of 'saving the phenomena' by calling attention to the way that *Alm.* 9.2 complements and strategically advances the philosophical position adopted in book one. The two observations (Sections 3-4 above) framing our close reading of the passage have demonstrated how the *apologia* of *Almagest* 9.2 appeals to a subtle but serious set of philosophical commitments that broadly justify Ptolemy's empirical and mathematical method and, in particular, explain the equant's alleged violation of the principle of uniform circular motion. Ptolemy's suggestion in 9.2 that mathematical experiment and observation are sufficient to modify the astronomer's ἀρχαί appeals to his revision of the Aristotelian hierarchy of sciences presented in *Alm.* 1.1. What is more, an allusion to Aristotelian doctrine in 9.2, if credible, would show Ptolemy to have not only revisited and extended the philosophical stance of 1.1, but also enriched it with additional methodological considerations that tell in his favor.[47]

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